A hash function \( h \) maps a set of \( n \) keys \( x_j \) into a set of integer values \( h(x_j) \) in the range \( 0 \leq h(x_j) \leq m-1 \) with duplicates allowed.

A hash function \( h \) is perfect if it has the additional property that:
for \( x_i \) and \( x_j \), _________________

A hash function \( h \) is minimal perfect if it is perfect and _______________

A hash function \( h \) is order preserving minimal perfect if it is minimal perfect and it has the property that if ____________ then _______________

What materials are needed to construct an OPMPHF for FTR?

1. _________________
   _________________

   Example: _________________

2. _________________

What is the procedure for producing an OPMPHF from the materials above?

__________________________

In the algorithm for constructing an OPMPHF given in the textbook, the edges of graph \( G \) are given by _________________ and the edges are labelled with _________________

If \( n=40,000 \) and \( m=2.5n \), the expected number of graphs that would have to be generated until the first acyclic graph is found is

__________________________